

# Alabama Statewide Math Contest - Round 1 Division Two

University of North Alabama

April 15, 2023

## Scoring

# Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive  
2 Bonus Points.

# Rules

## Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers  $\pi$  and  $e$  must be left as such.
4. Complex numbers must be put into  $a + bi$  form.

## Rules

# Rules

- Answers with radicals must be simplified. Denominators must be rationalized.
- Exponents should be positive.
- Answers involving trigonometric functions should be simplified as much as possible.
- $\log(x)$  means  $\log_{10}(x)$  and  $\ln(x)$  means  $\log_e(x)$ .
- The time limit for **all** problems is 2 minutes.

# Sample Problem # 1

## Sample Problem

RESET :

Solve for  $x$  in the equation

$$x^2 - 6x - 3 = 0$$

# Sample Problem

Answer:

## Sample Problem

Answer:  $3 + 2\sqrt{3}$  and  $3 - 2\sqrt{3}$ .



# Round 1

## Geometry

# Geometry Question # 1

# Geometry Question # 1

RESET :

A right circular cylinder with a height of 6 has a surface area of  $54\pi$ . Find its volume.

# Geometry Question # 1

Answer:

# Geometry Question # 1

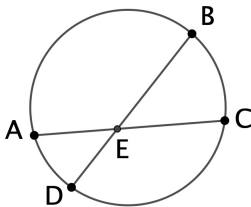
Answer:  $54\pi$

# Geometry Question # 2

## Geometry Question # 2

RESET :

Points  $A$ ,  $B$ ,  $C$  and  $D$  are on the circle, with secant lines  $\overline{AC}$  and  $\overline{BD}$  intersecting at point  $E$ . If  $m\widehat{BC} = 60^\circ$  and  $m\angle BEC = 50^\circ$ , find  $m\angle ECD$ , in degrees.



## Geometry Question # 2

Answer:



## Geometry Question # 2

Answer:  $20^\circ$

# Round 1

## Algebra II

# Algebra II Question # 3

## Algebra II Question # 3

RESET :

If  $x = 1$  is a solution to  $x^3 + 2x^2 - 31x + 28 = 0$ , find the larger of the other two solutions.

## Algebra II Question # 3

Answer:

## Algebra II Question # 3

Answer: 4

# Algebra II Question # 4

## Algebra II Question # 4

RESET :

If  $a$  and  $b$  are solutions to the equation  $(x - 1)(x + 1) = 8$ , find the value of  $ab$ .



# Algebra II Question # 4

Answer:

## Algebra II Question # 4

Answer:  $-9$

# Round 1

## Comprehensive Part 1

# Comprehensive Part 1

## Question # 5

## Comprehensive Part 1 Question # 5

RESET :

The graphs of  $y = x^2$  and  $y = 2x^2 - 3x + 2$  intersect at points  $A$  and  $B$ . What is the midpoint of segment  $\overline{AB}$ ?

# Comprehensive Part 1 Question # 5

Answer:

## Comprehensive Part 1 Question # 5

Answer:  $\left(\frac{3}{2}, \frac{5}{2}\right)$

# Comprehensive Part 1

## Question # 6



## Comprehensive Part 1 Question # 6

RESET :

If  $\sin \alpha = \frac{1}{2}$ , find  $1 - \tan^2 \alpha$ .

# Comprehensive Part 1 Question # 6

Answer:

# Comprehensive Part 1 Question # 6

Answer:  $\frac{2}{3}$

# Round 1

## Comprehensive Part 2

# Comprehensive Part 2

## Question # 7

## Comprehensive Part 2 Question # 7

RESET :

Let  $\star$  be defined by  $a \star b = a^2 + 2^b$ . If  $5 \star b = 41$ , what is  $b$ ?  
Provide your answer as an integer or simplified fraction.

## Comprehensive Part 2 Question # 7

Answer:

## Comprehensive Part 2 Question # 7

Answer: 4



# Comprehensive Part 2

## Question # 8

## Comprehensive Part 2 Question # 8

RESET :

Write the expression  $\frac{8i}{2-2i} + 3i - 4$  in the form  $a + bi$ , where  $i$  is the imaginary unit.

## Comprehensive Part 2 Question # 8

Answer:

## Comprehensive Part 2 Question # 8

Answer:  $-6 + 5i$

# Round 1

## Team

# Team Question # 9

## Team Question # 9

RESET :

Find the summation of

Volume of a right square based pyramid with a height of 5 and a  
base side of length 3

+

Measure of an exterior angle in a regular pentagon

+

Radius of circle defined by  $x^2 + 6x + y^2 - 12y = 4$

## Team Question # 9

Answer:



## Team Question # 9

Answer: 94

# Team Question # 10

## Team Question # 10

RESET :

A sequence is defined by  $a_n = a_{n-1} + a_{n-2} + a_{n-3}$  for  $n \geq 4$ .  
Suppose  $a_4 = 20$ ,  $a_5 = 36$ , and  $a_7 = 121$ . What is  $a_3$ ?

## Team Question # 10

Answer:

## Team Question # 10

Answer: 9

# End of Round 1