

# Alabama Statewide Math Contest - Round 2 Division Two

University of North Alabama

April 15, 2023

## Scoring

# Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive  
2 Bonus Points.

# Rules

## Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers  $\pi$  and  $e$  must be left as such.
4. Complex numbers must be put into  $a + bi$  form.

## Rules

# Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8.  $\log(x)$  means  $\log_{10}(x)$  and  $\ln(x)$  means  $\log_e(x)$ .
9. The time limit for **all** problems is 2 minutes.

# Sample Problem # 1

## Sample Problem

RESET :

Solve for  $x$  in the equation

$$x^2 - 6x - 3 = 0$$

# Sample Problem

Answer:

## Sample Problem

Answer:  $3 + 2\sqrt{3}$ , and  $3 - 2\sqrt{3}$ .



# Round 2

## Geometry

# Geometry Question # 1

# Geometry Question # 1

RESET :

Regular hexagon  $ABCDEF$  has diagonals  $\overline{AD}$  and  $\overline{BE}$  meeting at a point  $G$ . What is  $m\angle BGD$ , in degrees?

# Geometry Question # 1

Answer:

# Geometry Question # 1

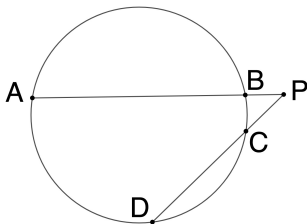
Answer:  $120^\circ$

# Geometry Question # 2

## Geometry Question # 2

RESET :

Points  $A$ ,  $B$ ,  $C$ , and  $D$  are on the circle, and  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$  intersect at point  $P$ . If  $BP = 2$ ,  $CP = 3$ , and  $DC = 5$ , what is  $AB$ ?



## Geometry Question # 2

Answer:



## Geometry Question # 2

Answer: 10

# Round 2

## Algebra II

# Algebra II Question # 3

## Algebra II Question # 3

RESET :

Find all solutions to  $\sqrt{x^2 - 4} + 2 = 5$ .

## Algebra II Question # 3

Answer:

## Algebra II Question # 3

Answer:  $\sqrt{13}$  and  $-\sqrt{13}$

# Algebra II Question # 4

## Algebra II Question # 4

RESET :

In the following matrix multiplication calculation, find the average value of variables  $A$ ,  $B$ ,  $C$ , and  $D$ .

$$\begin{bmatrix} -4 & 7 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$



# Algebra II Question # 4

Answer:

## Algebra II Question # 4

Answer: 14

# Round 2

## Comprehensive Part 1

# Comprehensive Part 1

## Question # 5

## Comprehensive Part 1 Question # 5

RESET :

Find the units digit of  $2^{23}$ .

# Comprehensive Part 1 Question # 5

Answer:

# Comprehensive Part 1 Question # 5

Answer: 8

# Comprehensive Part 1

## Question # 6



## Comprehensive Part 1 Question # 6

RESET :

Find the value of  $x$  for which  $\log_3(x + 1) - \log_3(x) = 1$ .

# Comprehensive Part 1 Question # 6

Answer:

# Comprehensive Part 1 Question # 6

Answer:  $\frac{1}{2}$

# Round 2

## Comprehensive Part 2

# Comprehensive Part 2

## Question # 7

## Comprehensive Part 2 Question # 7

RESET :

Let  $f(x)$  be the piecewise function defined below. Find the sum of all solutions for which  $f(x) = 7$ .

$$f(x) = \begin{cases} |x + 4| & \text{for } x \leq 1 \\ x^2 + 6x & \text{for } 1 < x < 6 \\ 2x - 5 & \text{for } x \geq 6 \end{cases}$$

# Comprehensive Part 2 Question # 7

Answer:

## Comprehensive Part 2 Question # 7

Answer:  $-5$



# Comprehensive Part 2

## Question # 8

## Comprehensive Part 2 Question # 8

RESET :

A rhombus has a side length of 5 and an area of 20. Find the sine of the smallest interior angle of the rhombus.

## Comprehensive Part 2 Question # 8

Answer:

## Comprehensive Part 2 Question # 8

Answer:  $\frac{4}{5}$

# Round 2

## Team

# Team Question # 9

## Team Question # 9

RESET :

Find the distance between the two solutions to the system of equations

$$\begin{cases} x + 3y = y^2 - 4 \\ x + y = 11 \end{cases}$$

## Team Question # 9

Answer:



## Team Question # 9

Answer:  $8\sqrt{2}$

# Team Question # 10

## Team Question # 10

RESET :

An equilateral triangle is inscribed in a circle, which is inscribed in a square. If the area of the square is 8, what is the area of the equilateral triangle?

## Team Question # 10

Answer:

## Team Question # 10

Answer:  $\frac{3\sqrt{3}}{2}$

# End of Round 2