

# Alabama Statewide Math Contest - Round 4 Division Two

University of North Alabama

April 15, 2023

## Scoring

# Scoring

0:00 - 0:30	10 points
0:31 - 1:00	8 points
1:01 - 1:30	6 points
1:31 - 2:00	4 points

If the first person to answer is correct, they receive  
2 Bonus Points.

# Rules

## Rules

1. Answers must be in answer box provided to be counted. Units such as cm, in, etc. are **not** necessary.
2. Fractions must be reduced. Improper fractions are acceptable.
3. The numbers  $\pi$  and  $e$  must be left as such.
4. Complex numbers must be put into  $a + bi$  form.

## Rules

# Rules

5. Answers with radicals must be simplified. Denominators must be rationalized.
6. Exponents should be positive.
7. Answers involving trigonometric functions should be simplified as much as possible.
8.  $\log(x)$  means  $\log_{10}(x)$  and  $\ln(x)$  means  $\log_e(x)$ .
9. The time limit for **all** problems is 2 minutes.

# Sample Problem # 1

## Sample Problem

RESET :

Solve for  $x$  in the equation

$$x^2 - 6x - 3 = 0$$

# Sample Problem

Answer:

## Sample Problem

Answer:  $3 + 2\sqrt{3}$ , and  $3 - 2\sqrt{3}$ .



# Round 4

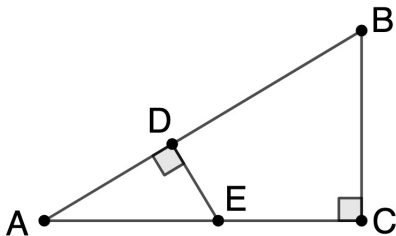
## Geometry

# Geometry Question # 1

## Geometry Question # 1

RESET :

In the figure below,  $AE = 5$ ,  $DE = 4$ ,  $BC = 12$ , and  $\angle ADE$  and  $\angle ACB$  are right angles. Find the length of  $\overline{EC}$ .



# Geometry Question # 1

Answer:

# Geometry Question # 1

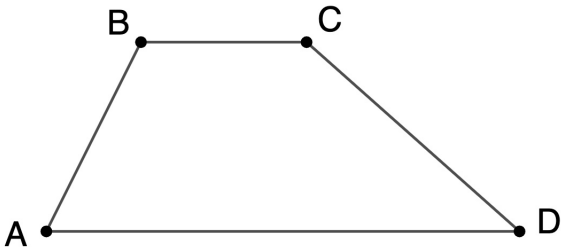
Answer: 4

# Geometry Question # 2

## Geometry Question # 2

RESET :

In trapezoid  $ABCD$ ,  $m\angle BAD = 60^\circ$ ,  $m\angle CDA = 30^\circ$ ,  $AB = 10$  and  $BC = 15$ . Find the area of the trapezoid.



## Geometry Question # 2

Answer:



## Geometry Question # 2

Answer:  $125\sqrt{3}$

# Round 4

## Algebra II

# Algebra II Question # 3

## Algebra II Question # 3

RESET :

If  $(x + y)^2 = 169$  and  $(x - y)^2 = 121$ , find  $xy$ .

# Algebra II Question # 3

Answer:

## Algebra II Question # 3

Answer: 12

# Algebra II Question # 4

## Algebra II Question # 4

RESET :

Find the smallest solution of the equation  $x^4 - 7x^2 + 12 = 0$ .



# Algebra II Question # 4

Answer:

## Algebra II Question # 4

Answer:  $-2$

# Round 4

## Comprehensive Part 1

# Comprehensive Part 1

## Question # 5

## Comprehensive Part 1 Question # 5

RESET :

Find the distance between the two solutions to  $x^2 + 3x + 3 = 0$  in the complex plane.

# Comprehensive Part 1 Question # 5

Answer:

# Comprehensive Part 1 Question # 5

Answer:  $\sqrt{3}$

# Comprehensive Part 1

## Question # 6



## Comprehensive Part 1 Question # 6

RESET :

A geometric sequence has  $a_1 = \frac{1}{2}$ ,  $a_2 = -1$ ,  $a_3 = 2$ , and  $a_4 = -4$ .  
Find  $a_{10}$ .

# Comprehensive Part 1 Question # 6

Answer:

## Comprehensive Part 1 Question # 6

Answer:  $-256$

# Round 4

## Comprehensive Part 2

# Comprehensive Part 2

## Question # 7

## Comprehensive Part 2 Question # 7

RESET :

The line  $y = 2x - 1$  intersects a circle at  $x = -3$  and  $x = 1$  in such a way that the secant line formed is a diameter of the circle. Find the area of the circle.

## Comprehensive Part 2 Question # 7

Answer:

## Comprehensive Part 2 Question # 7

Answer:  $20\pi$



# Comprehensive Part 2

## Question # 8

## Comprehensive Part 2 Question # 8

RESET :

Find the coefficient of the  $x$  term of the expansion of  $(2x + 3)^3$ .

## Comprehensive Part 2 Question # 8

Answer:

## Comprehensive Part 2 Question # 8

Answer: 54

# Round 4

## Team

# Team Question # 9

## Team Question # 9

RESET :

How many non-empty subsets of  $\{1, 2, 3, 4, 5, 6, 7\}$  consist of only odd integers?

## Team Question # 9

Answer:



## Team Question # 9

Answer: 15

# Team Question # 10

## Team Question # 10

RESET :

Find the value of

$$81^{1/4} + \cos^2\left(\frac{19\pi}{4}\right) + i^{10} + \log_{121} 11$$

## Team Question # 10

Answer:

## Team Question # 10

Answer: 3

# End of Round 4