The PRAXIS Series

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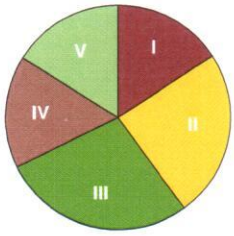
# Mathematics: Content Knowledge (0061/5061)

## Test at a Glance

| | | |
|---|---|---|
| Test Name | Mathematics: Content Knowledge | |
| Test Code | 0061 | 5061 |
| Time | 2 hours | 2 hours |
| Number of Questions | 50 | 50 |
| Format | Multiple-choice questions, graphing calculator required | Multiple-choice questions, graphing calculator required |
| Test Delivery | Paper delivered | Computer delivered |




| Content Categories | | Approximate Number of Questions | Approximate Percentage of Examination |
|---|---|---|---|
| I. | Algebra and Number Theory | 8 | 16% |
| II. | Measurement | 3 | 6% |
| | Geometry | 5 | 10% |
| | Trigonometry | 4 | 8% |
| III. | Functions | 8 | 16% |
| | Calculus | 6 | 12% |
| IV. | Data Analysis and Statistics | 5–6 | 10–12% |
| | Probability | 2–3 | 4–6% |
| V. | Matrix Algebra | 4–5 | 8–10% |
| | Discrete Mathematics | 3–4 | 6–8% |

| Process Categories | |
|---|---|
| Mathematical Problem Solving | Distributed Across Content Categories |
| Mathematical Reasoning and Proof | |
| Mathematical Connections | |
| Mathematical Representation | |
| Use of Technology | |

## About This Test

The Praxis Content Knowledge test in Mathematics is designed to assess the mathematical knowledge and competencies necessary for a beginning teacher of secondary school mathematics. Examinees have typically completed a bachelor's program with an emphasis in mathematics or mathematics education.

The examinee will be required to understand and work with mathematical concepts, to reason mathematically, to make conjectures, to see patterns, to justify statements using informal logical arguments, and to construct simple proofs. Additionally, the examinee will be expected to solve problems by integrating knowledge from different areas of mathematics, to use various representations of concepts, to solve problems that have several solution paths, and to develop mathematical models and use them to solve real-world problems. This test may contain some questions that will not count toward your score.

The test is not designed to be aligned with any particular school mathematics curriculum, but it is intended to be consistent with the recommendations of national studies on mathematics education, such as the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (2000) and the National Council for Accreditation of Teacher Education (NCATE) *Program Standards for Initial Preparation of Mathematics Teachers* (2003).

*Graphing calculators without QWERTY (typewriter) keyboards are required for this test*. Some questions will require the use of a calculator. The minimum capabilities required of the calculator are described in the section on graphing calculators. Because many test questions may be solved in more than one way, examinees should decide first how to solve each problem and then decide whether to use a calculator. On the test day, examinees should bring a calculator they are comfortable using.

Selected notations, formulas, and definitions are printed in the test book and are also listed on pages 8–10.

## Graphing Calculators

Examinees will be expected to bring to the examination a graphing calculator with the built-in capability to

1. produce the graph of a function within an arbitrary viewing window
2. find the zeros of a function
3. compute the derivative of a function numerically
4. compute definite integrals numerically

Computers, calculators with QWERTY (typewriter) keyboards, and electronic writing pads are NOT allowed.

Unacceptable machines include the following:

Powerbooks and portable/handheld computers

Pocket organizers

Electronic writing pads or pen-input/stylus-driven devices (e.g., Palm, PDA's, Casio Class Pad 300)

Devices with QWERTY keyboards (e.g., TI-92 PLUS, Voyage 200)

Cell-phone calculators

### PRAXIS
### Graphing Calculator Policy

Test administration staff will clear the memory of all graphing calculators both before and after the test administration.

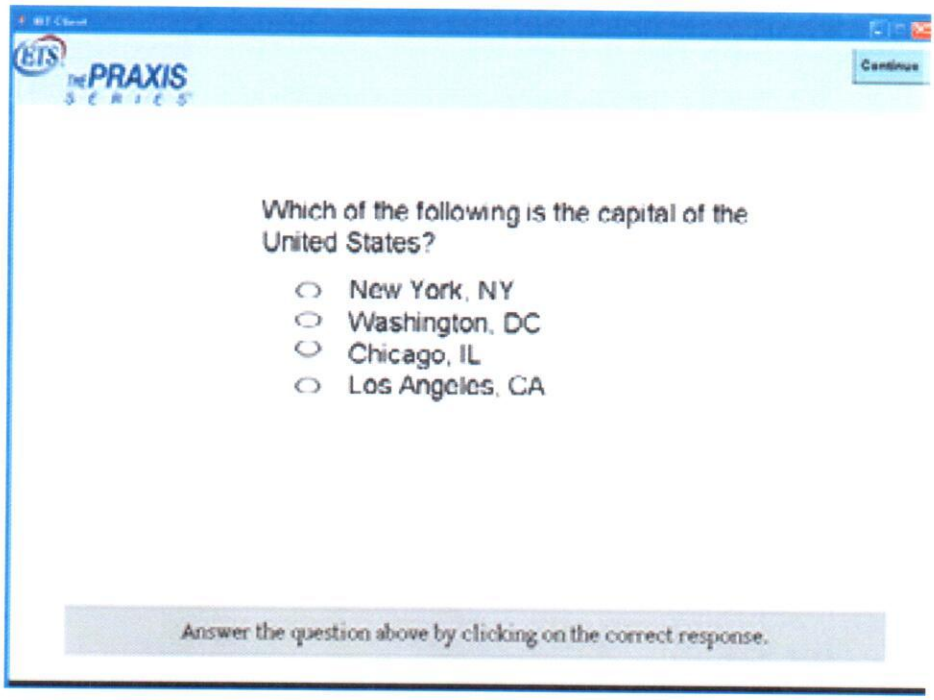
We recommend that you

- back up any important information in your calculator's memory, including applications, before arriving at the test site
- know how to clear the memory on the approved calculator that you plan to use during the test

Note: Instructions on how to back up and clear the memory of calculators can be found on various calculator Web sites.

This test is available via paper delivery or computer delivery; other than the delivery method, there is no difference between the tests. The test content is the same for both test codes.

To illustrate what the computer-delivered test looks like, the following sample question shows an actual screen used in a computer-delivered test.




Here is the same sample question as it would appear on a paper-delivered test:

Which of the following is the capital of the United States?

(A) New York, NY
(B) Washington, DC
(C) Chicago, IL
(D) Los Angeles, CA

For the purposes of this guide, sample questions are provided as they would appear in a paper-delivered test.

# Mathematics Content Descriptions — Basic

Representative descriptions of the topics covered in the basic content categories for the Content Knowledge and the Proofs, Models, and Problems tests follow. Because the assessments were designed to measure the ability to integrate knowledge of mathematics, answering any question may involve more than one competency and may involve competencies from more than one content area.

## Algebra and Number Theory 290-3-3-.13(2)(h)4.(i)

- Demonstrate an understanding of the structure of the natural, integer, rational, real, and complex number systems and the ability to perform the basic operations (+, − , × and ÷) on numbers in these systems
- Compare and contrast properties (e.g., closure, commutativity, associativity, distributivity) of number systems under various operations
- Demonstrate an understanding of the properties of counting numbers (e.g., prime, composite, prime factorization, even, odd, factors, multiples)
- Solve ratio, proportion, percent, and average (including arithmetic mean and weighted average) problems
- Work with algebraic expressions, formulas, and equations; add, subtract, and multiply polynomials; divide polynomials; add, subtract, multiply, and divide algebraic fractions; perform standard algebraic operations involving complex numbers, radicals, and exponents, including fractional and negative exponents
- Solve and graph systems of equations and inequalities, including those involving absolute value
- Interpret algebraic principles geometrically
- Recognize and use algebraic representations of lines, planes, conic sections, and spheres
- Solve problems in two and three dimensions (e.g., distance between two points, the coordinates of the midpoint of a line segment)

290-3-3-.13(2)(h)5.(i)
290-3-3-.13(2)(h)8.(i)
290-3-3-.13(2)(h)8.(iii)

290-3-3-.13(2)(h)4.(ii)
290-3-3-.13(2)(h)4.(iii)

## Measurement

- Make decisions about units and scales that are appropriate for problem situations involving measurement; use unit analysis
- Analyze precision, accuracy, and approximate error in measurement situations
- Apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations

## Geometry 290-3-3-.13(2)(h)6.(iii)

- Solve problems using relationships of parts of geometric figures (e.g., medians of triangles, inscribed angles in circles) and among geometric figures (e.g., congruence, similarity) in two and three dimensions
- Describe relationships among sets of special quadrilaterals, such as the square, rectangle, parallelogram, rhombus, and trapezoid
- Solve problems using the properties of triangles, quadrilaterals, polygons, circles, and parallel and perpendicular lines
- Solve problems using the properties of circles, including those involving inscribed angles, central angles, chords, radii, tangents, secants, arcs, and sectors
- Understand and apply the Pythagorean theorem and its converse
- Compute and reason about perimeter, area/surface area, or volume of two- or three-dimensional figures or of regions or solids that are combinations of these figures
- Solve problems involving reflections, rotations, and translations of geometric figures in the plane

## Trigonometry 290-3-3-.13(2)(h)7.(i)

- Define and use the six basic trigonometric functions using degree or radian measure of angles; know their graphs and be able to identify their periods, amplitudes, phase displacements or shifts, and asymptotes
- Apply the law of sines and the law of cosines
- Apply the formulas for the trigonometric functions of $\frac{x}{2}$, $2x$, $x$, $x + y$, and $x - y$; prove trigonometric identities
- Solve trigonometric equations and inequalities
- Convert between rectangular and polar coordinate systems

### Functions

- Demonstrate understanding of and ability to work with functions in various representations (e.g., graphs, tables, symbolic expressions, and verbal narratives) and to convert flexibly among them
- Find an appropriate family of functions to model particular phenomena (e.g., population growth, cooling, simple harmonic motion)
- Determine properties of functions and their graphs, such as domain, range, intercepts, symmetries, intervals of increase or decrease, discontinuities, and asymptotes
- Use the properties of trigonometric, exponential, logarithmic, polynomial, and rational functions to solve problems
- Determine the composition of two functions; find the inverse of a one-to-one function in simple cases and know why only one-to-one functions have inverses
- Interpret representations of functions of two variables, such as three-dimensional graphs, level curves, and tables

### Calculus

- Demonstrate understanding of what it means for a function to have a limit at a point; calculate limits of functions or determine that the limit does not exist; solve problems using the properties of limits
- Understand the derivative of a function as a limit, as the slope of a curve, and as a rate of change (e.g., velocity, acceleration, growth, decay)
- Show that a particular function is continuous; understand the relationship between continuity and differentiability
- Numerically approximate derivatives and integrals
- Use standard differentiation and integration techniques
- Analyze the behavior of a function (e.g., find relative maxima and minima, concavity); solve problems involving related rates; solve applied minima-maxima problems
- Demonstrate understanding of and ability to use the Mean Value Theorem and the Fundamental Theorem of Calculus
- Demonstrate understanding of integration as a limiting sum that can be used to compute area, volume, distance, or other accumulation processes
- Determine the limits of sequences and simple infinite series

290-3-3-.13(2)(w)7.(ii)

290-3-3-.13(2)(w)7.(iii)

290-3-3-.13(2)(h)9.(i)
290-3-3-.13(2)(h)9.(ii)

## Data Analysis and Statistics

- Organize data into a suitable form (e.g., construct a histogram and use it in the calculation of probabilities)
- Choose and apply appropriate measures of central tendency (e.g., population mean, sample mean, median, mode) and dispersion (e.g., range, population standard deviation, sample standard deviation, population variance, sample variance) to describe and compare data sets; recognize when to use sample statistics or population parameters
- Analyze data from specific situations to determine what type of function (e.g., linear, quadratic, exponential) would most likely model that particular phenomenon; use the regression feature of the calculator to determine curve of best fit; interpret the regression coefficients, correlation, and residuals in context

290-3-3-.13(2)(h)9.(iii)

- Understand and apply normal distributions and their characteristics (e.g., mean, standard deviation)
- Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference
- Understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each
- Know the characteristics of well-designed studies, including the role of randomization in surveys and experiments

## Probability

- Understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases
- Understand the concepts of conditional probability and independent events; understand how to compute the probability of a compound event
- Compute and interpret the expected value of random variables in simple cases (e.g., fair coins, expected winnings, expected profit)
- Use simulations to construct empirical probability distributions and to make informal inferences about the theoretical probability distribution

290-3-3-.13(2)(h)6.(ii)

## Matrix Algebra

- Understand vectors and matrices as systems that have some of the same properties as the real number system (e.g., identity, inverse, and commutativity under addition and multiplication)
- Scalar multiply, add, subtract, and multiply vectors and matrices; find inverses of matrices
- Use matrix techniques to solve systems of linear equations
- Use determinants to reason about inverses of matrices and solutions to systems of equations
- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, and matrices

## Discrete Mathematics

- Solve basic problems that involve counting techniques, including the multiplication principle, permutations, and combinations; use counting techniques to understand various situations (e.g., number of ways to order a set of objects, to choose a subcommittee from a committee, to visit $n$ cities)
- Find values of functions defined recursively and understand how recursion can be used to model various phenomena; translate between recursive and closed-form expressions for a function
- Determine whether a binary relation on a set is reflexive, symmetric, or transitive; determine whether a relation is an equivalence relation
- Use finite and infinite arithmetic and geometric sequences and series to model simple phenomena (e.g., compound interest, annuity, growth, decay)
- Understand the relationship between discrete and continuous representations and how they can be used to model various phenomena
- Use difference equations, vertex-edge graphs, trees, and networks to model and solve problems

290-3-3-.13(2)(h)8.(ii)

# Mathematical Process Categories

In addition to knowing and understanding the mathematics content explicitly described in the Content Descriptions section, entry-level mathematics teachers must also be able to think mathematically; that is, they must have an understanding of the ways in which mathematical content knowledge is acquired and used. Answering questions on this assessment may involve one or more of the processes described in the Process Categories below, and all of the processes may be applied to any of the content topics.

### Mathematical Problem Solving

- Solve problems that arise in mathematics and those involving mathematics in other contexts
- Build new mathematical knowledge through problem solving
- Apply and adapt a variety of appropriate strategies to solve problems

### Mathematical Reasoning and Proof 290-3-3-.13(2)(b)

- Select and use various types of reasoning and methods of proof
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs

### Mathematical Connections

- Recognize and use connections among mathematical ideas
- Apply mathematics in context outside of mathematics
- Demonstrate an understanding of how mathematical ideas interconnect and build on one another

290-3-3-.13(2)(d)1.(i)
290-3-3-.13(2)(d)1.(ii)
290-3-3-.13(2)(d)2.

290-3-3-.13(2)(e)1.
290-3-3-.13(2)(e)2.

### Mathematical Representation 290-3-3-.13(2)(e)3.

- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena
- Create and use representations to organize, record, and communicate mathematical ideas

### Use of Technology

- Use technology appropriately as a tool for problem solving and analysis
- Use technology as an aid to understanding mathematical ideas

290-3-3-.13(2)(w)10.(i)
290-3-3-.13(2)(w)10.(ii)