Written Round: February 28, 2026 at Regional Testing Sites

Second Round (by invitation): April 18, 2026 at Auburn University Montgomery

GEOMETRY EXAMINATION

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INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions are not arranged in order of difficulty. For each question, choose the best of the five options labeled A, B, C, D and E. Calculators are NOT permitted.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a paper with: all questions answered correctly earns a score of 250, all questions left blank earns a score of 50, and all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- $\log(x)$ means $\log_{10}(x)$ and $\ln(x)$ means $\log_e(x)$.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then $m\angle A$ is the measure of angle A in degrees.
- If A and B are points on a circle, then $\widehat{AB}$ is the arc between A and B.

- If A and B are points on a circle, then $m\widehat{AB}$ is the measure of $\widehat{AB}$ in degrees.
- If $\overline{AB} \cong \overline{CD}$, then $\overline{AB}$ and $\overline{CD}$ are congruent.
- If $\triangle ABC \cong \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are congruent.
- If $\triangle ABC \sim \triangle DEF$, then $\triangle ABC$ and $\triangle DEF$ are similar.
- If ℓ, m are two lines, then $\ell \perp m$ means ℓ and m are perpendicular.

Why Major in Mathematics?

What sorts of jobs can I get with a mathematics degree? Examples of occupational opportunities available to math majors:

- Market Research Analyst
- Air Traffic Controller
- Climate Analyst
- Estimator
- Research Scientist
- Computer Programmer

- Cryptanalyst
- Professor
- Pollster
- Population Ecologist
- Operations Research
- Data Analysis

- Mathematician
- Meteorologist
- Medical Doctor
- Lawyer
- Actuary
- Statistician

Where can I work? What sorts of companies hire mathematicians? Well just to name a few...

- **U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.
- **Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.
- **Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.
- **Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.
- **Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.
- **Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, Northrop Grumman, and Lockheed Martin.
- **Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).
- **Financial service and investment manangement firms** such as Citibank, Morgan Stanley, and Prudential.

A Mathematics Major isn't just for those wanting to be Mathematicians!

- The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)
- Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems.

1. The degree measures of the four angles of a convex quadrilateral are an arithmetic progression. If the smallest angle has a degree measure of $45°$, what is the degree measure of the largest angle?

(**A**) $105°$ (**B**) $115°$ (**C**) $125°$ (**D**) $135°$ (**E**) None of these

Solution: The arithmetic progression is

$$45° + (45° + d) + (45° + 2d) + (45° + 3d) = 360°.$$

$$6d + 180° = 360°$$

$$6d = 180°$$

$$d = 30°$$

Largest angle is
$$45° + 3d = 45° + 3(30°) = 45° + 90° = 135°.$$

Answer: D) $135°$

2. The planes containing the faces of a cube divide the space into several 3-dimensional regions. Find the number of regions.

(**A**) 8 (**B**) 27 (**C**) 32 (**D**) 40 (**E**) None of these

Solution: Each pair of parallel planes divides the space into three parts. The faces of the cube determine 3 pairs of parallel planes which are not mutually parallel. Therefore, a cube divides the space into $(3 \times 3 \times 3 = 27)$ *regions.*

Answer: B) 27

3. Two complementary angles have measures $2x$ and $3x - 10$. Find the supplement of the larger angle.

(**A**) $130°$ (**B**) $135°$ (**C**) $140°$ (**D**) $145°$ (**E**) None of these

Solution:

Since the two angles are complementary,

$$2x + 3x - 10 = 90.$$

Then
$$5x - 10 = 90 \quad \Rightarrow \quad x = 20.$$

So the smaller angle is
$$2x = 40°,$$

and the larger angle is
$$3x - 10 = 50°.$$

The supplement of the larger angle is
$$180° - 50° = 130°.$$

Answer: A) $130°$

4. The length of the shorter side of a rectangle is 2 units. The length of each diagonal is 4 units. Find the measure of the acute angle between the diagonals.

(**A**) 30°   (**B**) 45°   (**C**) 60°   (**D**) 72°   (**E**) None of these

Solution:

Suppose the vertices of the rectangle are labeled A, B, C, and D in such a way that AB is the shorter side. Let O be the intersection point of the diagonals. Then triangle AOB is equilateral. So the measure of the acute angle is $60°$.

Answer: C) $60°$

5. A regular polygon has an interior angle that measures $160°$, and a side of which is 14 units long. What is the perimeter of the regular polygon?

(**A**) 144   (**B**) 196   (**C**) 210   (**D**) 252   (**E**) None of these

Solution:

The number of sides of the regular polygon is found as follows:

$$160 = \frac{180(n-2)}{n}$$

$$360 = 20n \quad \Rightarrow \quad n = 18.$$

Then the perimeter of the polygon is
$$18 \times 14 = 252 \text{ units.}$$

Answer: D) 252

6. A triangle with sides 6, 8, and 10 has its shortest side doubled in length, while the other two sides remain the same. What is the area of the new triangle?

(**A**) 30   (**B**) 40   (**C**) $15\sqrt{7}$ (**D**) $8\sqrt{30}$ (**E**) None of these

Solution: The new triangle will have side lengths 8, 10, and 12. The perimeter is 30, and the semiperimeter is 15.
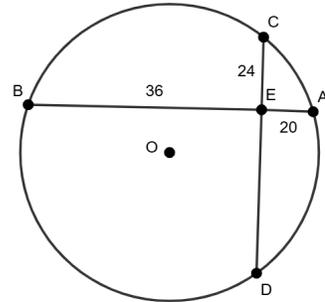
Using Heron's formula:
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15(15-8)(15-10)(15-12)} = \sqrt{15 \cdot 7 \cdot 5 \cdot 3} = 15\sqrt{7}.$$

Answer: C) $15\sqrt{7}$

7. In circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at right angles at E. If $AE = 20$, $EB = 36$, and $CE = 24$, determine the measure of DE.

    (A) 30   **(B)** 38   **(C)** 42   **(D)** 46   **(E)** None of these

Solution: In the figure shown, we see that

$$EB \cdot AE = CE \cdot DE.$$

Let $x = DE$. Then

$$36 \cdot 20 = 24 \cdot x$$
$$720 = 24x$$
$$x = 30.$$

Therefore,

$$DE = 30.$$

Answer: A) 30

8. When the volume of a spherical balloon is multiplied by 3, by what factor does its surface area increase?

    (A) $3^{4/3}$ **(B)** $3^{1/2}$ **(C)** $3^{1/3}$ **(D)** 3^{-1} **(E)** None of these

Solution: When the radius is increased by a factor x, the volume is increased by x^3 and surface area by x^2. So if $x^3 = 3$, then

$$x = 3^{1/3} \quad \text{and} \quad x^2 = 3^{2/3}.$$

Answer: E) None of these

9. Let u be a positive real number. The tip of a right circular cone, with base of radius $10u$ and height of $8u$, is removed with a horizontal slice of $3u$ below the tip. Find the radius of the circle that is formed. (The figure is a vertical cross-section of the cone.)

    (A) $\dfrac{11}{3}u$ **(B)** $5u$ **(C)** $\dfrac{13}{5}u$

    (D) $\dfrac{15}{4}u$ **(E)** None of these
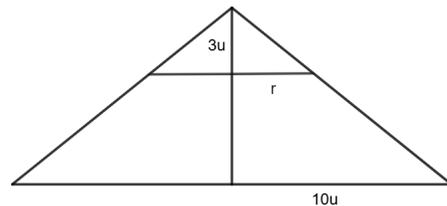
Solution: Let r be the radius. Using similar triangles, we have the proportion

$$\frac{r}{10} = \frac{3}{8}.$$

$$r = \frac{15}{4}u \quad \text{in reduced form.}$$

Answer: D) $\dfrac{15}{4}u$

10. If one of the interior angles of a rhombus is $60°$, then find the ratio of the area of the rhombus to the area of a circle inscribed in the rhombus.

(**A**) $\dfrac{6\sqrt{2}}{5\pi}$ (**B**) $\dfrac{8\sqrt{3}}{3\pi}$ (**C**) $\dfrac{7\sqrt{12}}{4\pi}$ (**D**) $\dfrac{11\sqrt{3}}{3\pi}$ (**E**) None of these

Solution: Label the vertices of the rhombus as $ABCD$. Draw a diagonal from A to C. Let M be the midpoint of AC. The perpendicular distance from M to each of the sides of the rhombus is equal to $\frac{1}{2}AB\sin 60°$, which is the radius of the circle.

So the area of the rhombus is
$$(\tfrac{1}{2}AB\sin 60°)^2.$$

Therefore the ratio is

$$\frac{\text{Area of Rhombus}}{\text{Area of Circle}} = \frac{AB^2 \sin 60°}{\pi(\frac{1}{2}AB\sin 60°)^2} = \frac{4}{\pi \sin 60°} = \frac{8}{\pi\sqrt{3}} = \frac{8\sqrt{3}}{3\pi}.$$

Answer: B) $\dfrac{8\sqrt{3}}{3\pi}$

11. Which is the equation of a circle with radius 8 and whose center is the intersection of the lines $6x + 2y = 2$ and $-4x - 6y = 8$?

(**A**) $x^2 + y^2 - 2x + 4y - 59 = 0$ (**B**) $x^2 + y^2 - 4x - 6y - 1 = 0$

(**C**) $x^2 + y^2 - 2x + 4y + 59 = 0$ (**D**) $x^2 - y^2 + 4x + y + 61 = 0$ (**E**) None of these

Solution: Solving the given system we obtain the ordered pair $(1, -2)$. The standard equation is
$$(x-1)^2 + (y+2)^2 = 64.$$

The general equation is
$$x^2 + y^2 - 2x + 4y - 59 = 0.$$

Answer: A) $x^2 + y^2 - 2x + 4y - 59 = 0$

12. A right circular cone has a base area of 25π, a lateral surface area of 65π, and a total surface area of 90π. What is its volume?

(**A**) 80π (**B**) 90π (**C**) 100π (**D**) 110π (**E**) None of these

Solution: For a right circular cone, the base area is πr^2, so
$$\pi r^2 = 25\pi \Rightarrow r = 5.$$

The lateral surface area is $r\pi s = 65\pi$, so
$$5\pi s = 65\pi \Rightarrow s = 13.$$

Now
$$r^2 + h^2 = s^2,$$
$$25 + h^2 = 169 \Rightarrow h^2 = 144 \Rightarrow h = 12.$$

The volume is
$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}(25\pi)(12) = 100\pi.$$

Answer: C) 100π

13. The area of a circle is eight times the circumference. Find the area of this circle.

    (**A**) 144π (**B**) 196π (**C**) 225π (**D**) 256π (**E**) None of these

Solution: Let r be the radius. The area is πr^2 and the circumference is $2\pi r$. Given:

$$\pi r^2 = 8(2\pi r).$$

$$\pi r^2 = 16\pi r \Rightarrow r(r - 16) = 0.$$

Thus $r = 16$. Then the area is
$$\pi r^2 = 256\pi.$$

Answer: D) 256π

14. An acute angle is formed by two lines of slope 1 and 7. What is the positive slope of the line which bisects this angle?

    (**A**) $\dfrac{1}{2}$ (**B**) 1   (**C**) $\dfrac{4}{3}$ (**D**) 2   (**E**) None of these

Solution: Using the formula for the tangent of the difference of two angles,

$$\frac{x - 1}{1 + x} = \frac{7 - x}{1 + x}$$

where x is the desired slope.

Rewriting and solving:
$$7x - 6x - 1 = -x^2 + 6x + 7$$

which reduces to
$$8x^2 - 12x - 8 = 0.$$

This gives a positive solution of 2.

Answer: D) 2

15. Let A, B, C denote the vertices of a triangle with area 10. Let point D be on side $\overline{AB}$, point E on side $\overline{BC}$, and point F on side $\overline{CA}$ with $AD = 2$ and $DB = 3$. The area of $\triangle ABE$ and the area of the quadrilateral $BEFD$ are the same. What is the area of $\triangle DEB$?

    (**A**) 4   (**B**) 5.5   (**C**) 6.5   (**D**) 7   (**E**) None of these

Solution: First note that $\triangle AFD$ and $\triangle AFE$ have the same area, so ED is parallel to AF. Since $\triangle DEB$ and $\triangle ACB$ are similar,
$$\frac{BC}{EC} = \frac{3}{5}.$$

Thus the area of
$$\triangle DEB = \frac{3}{5}(\text{Area of } \triangle ABC) = \frac{3}{5}(10) = 6.$$

Answer: E) None of these

16. The edges of a regular tetrahedron with vertices A, B, C, D each have length one. Find the smallest possible distance between a pair of points P and Q, where P is on edge $\overline{AB}$ and Q is on edge $\overline{CD}$.

(**A**) $\dfrac{\sqrt{2}}{2}$ (**B**) $\dfrac{1}{2}$ (**C**) $\dfrac{3}{4}$ (**D**) $\dfrac{\sqrt{3}}{2}$ (**E**) None of these

Solution: The distance between P and Q is minimized when they are the midpoints of AB and CD, respectively. The segments PC and PD are altitudes of the equilateral triangles ABC and ABD. Thus $PC = PD = \dfrac{\sqrt{3}}{2}$. Applying the Pythagorean theorem to $\triangle CPQ$ with $PQ = x$,

$$x^2 + \left(\frac{1}{2}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2,$$

$$x^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}.$$

Thus the smallest distance is $\dfrac{\sqrt{2}}{2}$.

Answer: A) $\dfrac{\sqrt{2}}{2}$

17. The length of a rectangle is tripled and the width is quadrupled. If the area of the original rectangle is $64u^2$, then what is the area of the new rectangle?

(**A**) $540u^2$ (**B**) $676u^2$ (**C**) $768u^2$ (**D**) $834u^2$ (**E**) None of these

Solution: The original area is $A = LW$. The new area is $(3L)(4W) = 12LW = 12A$. So the area of the new rectangle is $12 \cdot 64u^2 = 768u^2$.

Answer: C) $768u^2$

18. In $\triangle ABC$, the area is numerically equal to the perimeter. What is the radius of the inscribed circle?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) None of these

Solution: In the figure of $\triangle ABC$ (not shown), we have

$$[\triangle ABO] = \frac{1}{2}AB \cdot h = \frac{1}{2}AB \cdot r,$$

$$[\triangle BCO] = \frac{1}{2}BC \cdot r,$$

$$[\triangle CAO] = \frac{1}{2}CA \cdot r.$$

Thus

$$[\triangle ABC] = \frac{r}{2}(AB + BC + CA) = \frac{rS}{2},$$

where S is the perimeter. Since $S = [\triangle ABC]$, we get

$$S = \frac{rS}{2}.$$

Thus $r = 2$. **Answer:** B) 2

19. A solid regular octahedron has volume $\sqrt{6}$ cubic inches. Determine the surface area of the octahedron.

(**A**) $2\sqrt{2}$ sq. in. (**B**) $2\sqrt{3}$ sq. in. (**C**) $4\sqrt{2}$ sq. in. (**D**) $4\sqrt{3}$ sq. in. (**E**) None of these

Solution: A regular octahedron is two square pyramids joined together. The volume of one pyramid with side s is $\dfrac{s^2 h}{3}$, where h is the altitude. Using the Pythagorean theorem twice, the altitude is $\dfrac{s\sqrt{2}}{2}$.

$$\text{Volume} = 2\left(\frac{s^2}{3} \cdot \frac{s\sqrt{2}}{2}\right) = \frac{s^3\sqrt{2}}{3}.$$

Given volume $\sqrt{6}$,

$$\frac{s^3\sqrt{2}}{3} = \sqrt{6} \quad \Rightarrow \quad s^3 = 3\sqrt{3}.$$

Each face is equilateral with area $\dfrac{\sqrt{3}}{4}s^2$, and there are 8 faces, so
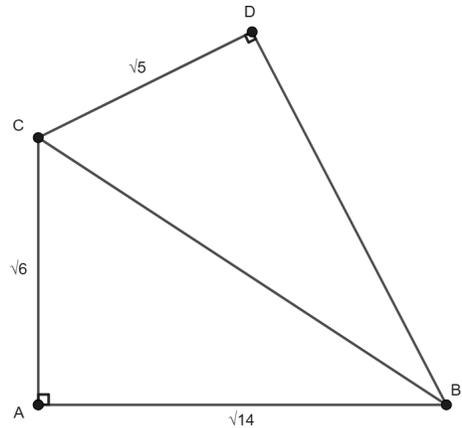
$$\text{Surface Area} = 8 \cdot \frac{\sqrt{3}}{4}s^2 = 2\sqrt{3}\,s^2.$$

Substituting $s^2 = \sqrt[3]{9}$ gives the surface area $6\sqrt{3}$ sq. in.

Answer: E) $6\sqrt{3}$

20. In the given figure, find the length of $\overline{BD}$. (The angles at A and D are right angles and the lengths are $AB = \sqrt{14}$, $AC = \sqrt{6}$, and $CD = \sqrt{5}$.)

(**A**) $\sqrt{10}$ (**B**) $\sqrt{15}$ (**C**) $2\sqrt{10}$

(**D**) $4\sqrt{5}$ (**E**) None of these

Solution: Using the Pythagorean theorem in $\triangle ABC$,

$$AC^2 + AB^2 = BC^2,$$

$$16^2 + 114^2 = BC^2 \Rightarrow BC = 2\sqrt{15}.$$
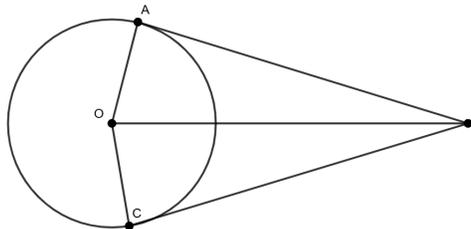
Again in $\triangle BCD$,

$$CD^2 + BD^2 = BC^2,$$

$$15^2 + BD^2 = (2\sqrt{15})^2 = 60,$$

$$BD^2 = 15 \Rightarrow BD = \sqrt{15}.$$

Answer: B) $\sqrt{15}$

21. A circle with center at point O has radius 1. A point B is a distance of 3 away from point O. The tangents to the circle from B touch the circle at points A and C. What is the area of the quadrilateral $OABC$?

(**A**) 2   (**B**) 3   (**C**) $2\sqrt{2}$

(**D**) $\sqrt{10}$ (**E**) None of these

Solution: The tangents form right angles with the radii to A and C. Thus each right triangle has one leg 1 and hypotenuse 3, so the other leg is

$$\sqrt{3^2 - 1^2} = \sqrt{8}.$$

The area of $OABC$ is

$$2\left(\tfrac{1}{2}(b)(h)\right) = 2(\tfrac{1}{2} \cdot 1 \cdot \sqrt{8}) = 2\sqrt{2}.$$

Answer: C) $2\sqrt{2}$

22. In $\triangle ABC$, $AB = 20$, $BC = 7$, and $CA = 15$. Side $\overline{BC}$ is extended to point D so that $\triangle DAB$ is similar to $\triangle DCA$. Find the perimeter of $\triangle DAB$.

(**A**) 41   (**B**) 44   (**C**) 46   (**D**) 48   (**E**) None of these

Solution: Since the triangles are similar,

$$\frac{DC}{DA} = \frac{AC}{AB} = \frac{15}{20} = \frac{3}{4}.$$

Thus $DC = \dfrac{3}{4}DA$ and $DB = \dfrac{5}{4}DA$. But $DB - DC = BC = 7$, so

$$\frac{5}{4}DA - \frac{3}{4}DA = DA = 7.$$

Then $DA = 12$, $DB = 16$. Hence the perimeter of $\triangle DAB$ is

$$DA + AB + DB = 12 + 20 + 16 = 48.$$

Answer: D) 48

23. Suppose $\triangle ABC$ has area $\dfrac{\sqrt{3} - 1}{2}$, $AB = \sqrt{3} - 1$, $AC = 2$, and angle $\angle CAB$ is acute. Determine the measure of angle $\angle ACB$ (in degrees).

(**A**) $15°$ (**B**) $18°$ (**C**) $20°$ (**D**) $30°$ (**E**) None of these

Solution: Let D be the point on AB such that $CD \perp AB$. Since the base and area give height $CD = 1$, and $AD = \sqrt{3}$, angle $CAD = 30°$. Since $AD = \sqrt{3}$ and $AB = \sqrt{3} - 1$, we get $BD = 1$. Considering the right triangle BDC, angle $BCD = 45°$. Thus

$$\angle ACB = 15°.$$

Answer: A) $15°$

24. Let C_1 and C_2 be concentric circles with C_1 being the larger. A chord of C_1 tangent to C_2 has length 28. What is the area of the region which lies between C_1 and C_2?

(**A**) 121π (**B**) 144π (**C**) 196π (**D**) 225π (**E**) None of these

Solution: Let the radii be R and r. Because the chord is tangent to C_2, the distance from the center to the chord is r, so
$$R^2 - r^2 = 14^2 = 196.$$
Thus the required area is
$$\pi(R^2 - r^2) = 196\pi.$$

Answer: C) 196π

25. An equilateral triangle is inscribed in a circle with radius 2. Find the perimeter of the equilateral triangle.

   (**A**) $4\sqrt{3}$ (**B**) $6\sqrt{3}$ (**C**) $8\sqrt{3}$ (**D**) $10\sqrt{3}$ (**E**) None of these

Solution: The central angle is $120°$ and half-angle is $60°$. Using $\cos 30° = \dfrac{\sqrt{3}}{2}$ gives half-side
$$\frac{s}{2} = \frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3}.$$
Thus $s = 2\sqrt{3}$ and the perimeter is
$$3s = 6\sqrt{3}.$$

Answer: B) $6\sqrt{3}$

26. The interior angles of a convex polygon of 9 sides are in arithmetic progression. If the smallest interior angle is $112°$, find the difference between the largest interior angle and the smallest.

   (**A**) 46   (**B**) 49   (**C**) 52   (**D**) 55   (**E**) None of these

Solution: The sum is $1260°$. The average angle is $140°$, so the fourth term is $140°$. Thus $d = 28°$. Largest angle is $112 + 8d = 168°$. Difference:
$$168 - 112 = 56.$$

Answer: E) None of these

27. A cylinder with radius r and height h has volume 1. If the total surface area is 128, then find
$$\left(\frac{1}{h} + \frac{1}{r}\right)^{1/6}.$$

   (**A**) 2   (**B**) $2\sqrt{2}$ (**C**) 4   (**D**) $4\sqrt{2}$ (**E**) None of these

Solution: From $\pi r^2 h = 1$ we get
$$h = \frac{1}{\pi r^2} \implies \pi r^2 = \frac{1}{h}.$$
Surface area:
$$2\pi r h + 2\pi r^2 = 128.$$
Substituting:
$$\frac{2}{r} + \frac{2}{h} = 128.$$
Thus
$$\frac{1}{r} + \frac{1}{h} = 64.$$
Therefore
$$\left(\frac{r}{h} + \frac{h}{r}\right)^{1/6} = \frac{6}{\sqrt[6]{64}} = 2.$$

Answer: A) 2

28. The line $x + y = k$ is tangent to the circle $x^2 + y^2 = 1$. Find the value of k.

(**A**) 1   (**B**) $\dfrac{3\pi}{4}$ (**C**) $\sqrt{2}$ (**D**) $\dfrac{\sqrt{2}}{2}$ (**E**) None of these

Solution: A tangent has distance from center equal to radius:
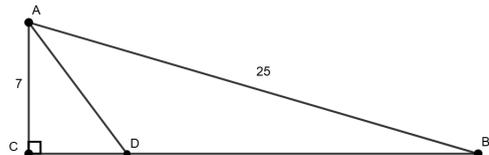$$\frac{|k|}{\sqrt{1^2 + 1^2}} = 1.$$
Thus
$$\frac{|k|}{\sqrt{2}} = 1 \quad \Rightarrow \quad |k| = \sqrt{2}.$$
Positive value gives $k = \sqrt{2}$.

Answer: C) $\sqrt{2}$

29. In $\triangle BCA$, angle C is a right angle and $\overline{AD}$ is a bisector of angle $\angle BAC$. Given that $AC = 7$ and $AB = 25$, find the length of segment $\overline{CD}$.

(**A**) $\dfrac{21}{4}$ (**B**) $\dfrac{24}{7}$ (**C**) 12

(**D**) $\dfrac{75}{7}$ (**E**) None of these

Solution: By the angle bisector theorem and area decomposition,
$$CD = \frac{21}{4}.$$

Answer: A) $\dfrac{21}{4}$

30. Let (h, k) denote the center and r the radius of the circle given by
$$x^2 + 2x + y^2 - 4y = 4.$$

What is the sum $h^2 + k^2 + r^2$?

(**A**) 8   (**B**) 10   (**C**) 12   (**D**) 14   (**E**) None of these

Solution: Completing the square:
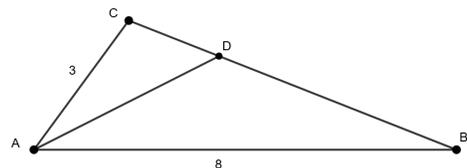$$(x + 1)^2 + (y - 2)^2 = 3^2.$$
Thus $h = -1$, $k = 2$, $r = 3$. So
$$h^2 + k^2 + r^2 = 1 + 4 + 9 = 14.$$

Answer: D) 14

31. In $\triangle ABC$, $AB = 8$, $AC = 3$, and $m\angle BAC = 60°$. If $\overline{AD}$ bisects $\angle BAC$, find the length of $\overline{AD}$.

(**A**) $\dfrac{17}{11}$ (**B**) $\dfrac{19}{13}$ (**C**) $\dfrac{21}{11}$

(**D**) $\dfrac{24}{17}$ (**E**) None of these

Solution: By the Law of Cosines,

$$BC = \sqrt{3^2 + 8^2 - 2 \cdot 3 \cdot 8 \cos 60°} = \sqrt{9 + 64 - 24} = 7.$$

Since an angle bisector divides the opposite side proportionally,

$$CD = \frac{3}{3+8} \cdot 7 = \frac{21}{11}.$$

Thus $AD = \dfrac{21}{11}$.

Answer: C) $\dfrac{21}{11}$

32. The area of a circle circumscribed about a regular hexagon is 200π. What is the area of the hexagon?

(**A**) $300\sqrt{3}$ (**B**) $450\sqrt{3}$ (**C**) $525\sqrt{3}$ (**D**) $600\sqrt{3}$ (**E**) None of these

Solution:

$$200\pi = \pi r^2 \quad \Rightarrow \quad r = 10\sqrt{2}.$$

A regular hexagon consists of six equilateral triangles of side r. Each has area $\frac{\sqrt{3}}{4} r^2 = \frac{\sqrt{3}}{4}(200) = 50\sqrt{3}$. So the hexagon's area is

$$6 \cdot 50\sqrt{3} = 300\sqrt{3}.$$

Answer: A) $300\sqrt{3}$

33. Let A through F be the vertices of a regular hexagon listed clockwise with side length 1. Find the area of $\triangle ACE$.

(**A**) $\dfrac{3\sqrt{3}}{4}$ (**B**) $\dfrac{3\sqrt{3}}{5}$ (**C**) $\dfrac{3\sqrt{3}}{8}$ (**D**) $\dfrac{3\sqrt{3}}{10}$ (**E**) None of these

Solution: Let side $AB = 1$. The area of the hexagon is $\dfrac{3\sqrt{3}}{2}$ and $|AC| = \sqrt{3}$. Thus the area of $\triangle ACE$ is

$$\frac{3\sqrt{3}}{4}.$$

Answer: A) $\dfrac{3\sqrt{3}}{4}$

34. Find the perimeter of the triangle whose vertices are $(1, -1)$, $(5, -1)$, $(3, 8)$.

(**A**) $\sqrt{85} + 2$ (**B**) $2\sqrt{21} + 4$ (**C**) $\sqrt{63} + 2$ (**D**) $2\sqrt{85} + 4$ (**E**) None of these

Solution:

$$D_{AB} = 4, \quad D_{AC} = \sqrt{85}, \quad D_{BC} = \sqrt{85}.$$

Thus the perimeter is

$$4 + \sqrt{85} + \sqrt{85} = 2\sqrt{85} + 4.$$

Answer: D) $2\sqrt{85} + 4$

35. Two similar polygons have corresponding sides 16 and 24. If the area of the larger polygon is 180, find the area of the smaller polygon.

(**A**) 60   (**B**) 70   (**C**) 80   (**D**) 90   (**E**) None of these

Solution:

$$\frac{S}{180} = \left(\frac{16}{24}\right)^2 = \frac{256}{576}.$$

Thus

$$S = 80.$$

Answer: C) 80

36. The sides of a triangle are 14, 16, and 18. Find the length of the longest altitude.

(**A**) $\dfrac{48\sqrt{5}}{7}$ (**B**) $4\sqrt{5}$ (**C**) $\dfrac{24\sqrt{5}}{7}$ (**D**) $8\sqrt{5}$ (**E**) None of these

Solution: Heron's formula:

$$s = 24, \quad A = \sqrt{24 \cdot 10 \cdot 8 \cdot 6} = 48\sqrt{5}.$$

Using smallest side 14 as base,

$$h = \frac{2A}{14} = \frac{48\sqrt{5}}{7}.$$

Answer: A) $\dfrac{48\sqrt{5}}{7}$

37. Determine the volume of a cube with surface area $726u^2$.

(**A**) $343u^3$ (**B**) $512u^3$ (**C**) $729u^3$ (**D**) $1000u^3$ (**E**) None of these

Solution:

$$6s^2 = 726 \quad \Rightarrow \quad s^2 = 121, \ s = 11.$$

Thus

$$s^3 = 1331u^3.$$

Answer: E) None of these

38. A parallelogram has diagonals of lengths 28 and 32. Let the lengths of two adjacent sides be a and b. Find $a^2 + b^2$.

(**A**) 708 (**B**) 842 (**C**) 904 (**D**) 1126 (**E**) None of these

Solution:

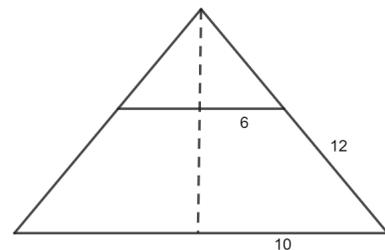$$2a^2 + 2b^2 = 28^2 + 32^2 = 784 + 1024 = 1808.$$

Thus

$$a^2 + b^2 = 904.$$

Answer: C) 904

39. A frustum of a cone has top radius 6, bottom radius 10, and slanted height between bases 12. What is the volume of the full cone?

(**A**) $\dfrac{400\pi\sqrt{2}}{3}$ (**B**) $\dfrac{2000\pi\sqrt{2}}{3}$ (**C**) $\dfrac{2600\pi\sqrt{2}}{3}$

(**D**) $\dfrac{2900\pi\sqrt{2}}{3}$ (**E**) None of these

Solution: Using similarity:
$$\frac{6}{10} = \frac{10}{x+12} \quad \Rightarrow \quad x = 18.$$

Thus full slant height is 30. Altitude:
$$h = \sqrt{30^2 - 10^2} = \sqrt{800} = 20\sqrt{2}.$$

Volume:
$$V = \frac{1}{3}\pi(10^2)(20\sqrt{2}) = \frac{2000\sqrt{2}\,\pi}{3}.$$

Answer: B) $\dfrac{2000\pi\sqrt{2}}{3}$

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40. The common internal tangent to two circles is 7 and the common external tangent is 11. One radius is three less than the other. Find the radius of the smaller circle.

(**A**) 1   (**B**) 2   (**C**) 3   (**D**) 4   (**E**) None of these

Solution: Internal tangent gives:
$$(R+r)^2 + 49 = d^2.$$

External tangent gives:
$$(R-r)^2 + 121 = d^2.$$

Subtracting:
$$(R-r)^2 - (R+r)^2 = 49 - 121 = -72,$$
$$-4Rr = -72 \quad \Rightarrow \quad Rr = 18.$$

Given $R = r + 3$:
$$(r+3)r = 18,$$
$$r^2 + 3r - 18 = 0 \quad \Rightarrow \quad r = 3.$$

Answer: C) 3

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41. A circular pool has diameter 24. A walkway of width 2 ft surrounds it. Find the area of the walkway.

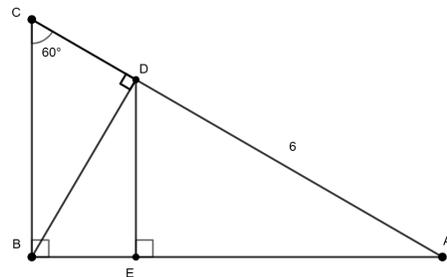(**A**) 32π (**B**) 38π (**C**) 46π (**D**) 52π (**E**) None of these

Solution: Pool radius $= 12$, area $= 144\pi$. Outer radius $= 14$, area $= 196\pi$. Walkway area:
$$196\pi - 144\pi = 52\pi.$$

Answer: D) 52π

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42. In right triangle $\triangle ABC$, $\overline{BD} \perp \overline{AC}$ and $\overline{DE} \perp \overline{AB}$. If $m\angle C = 60°$ and $DA = 6$, find the perimeter of $\triangle ABC$.

(**A**) $6\sqrt{2} + 12$ (**B**) $4\sqrt{3} + 12$ (**C**) $6\sqrt{3} + 12$

(**D**) $6\sqrt{2} + 12\sqrt{3}$ (**E**) None of these

Solution: From angle relations, $EA = 3\sqrt{3}$ and $DE = 3$. Thus
$$AB = 4\sqrt{3}, \quad BC = 4, \quad AC = 8.$$
Perimeter $= 4\sqrt{3} + 4 + 8 = 4\sqrt{3} + 12$.

Answer: B) $4\sqrt{3} + 12$

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43. A cone has height nine times its radius. If the volume is 375π, find the radius.

    (**A**) 3   (**B**) 5   (**C**) 7   (**D**) 9   (**E**) None of these

Solution: Height $h = 9r$.
$$375\pi = \frac{1}{3}\pi r^2(9r) = 3\pi r^3.$$
Thus $r^3 = 125$, so $r = 5$.

Answer: B) 5

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44. In rectangle $QUAD$, $QU = 15$ and $UA = 10$. A person P is standing inside the rectangle so that $QP = 9$ and $UP = 12$. How far is P from corner A?

    (**A**) 11   (**B**) 12   (**C**) 14   (**D**) 15   (**E**) None of these

Solution: In $\triangle QUP$, $9^2 + 12^2 = 15^2$, so $m\angle QUP = 90°$. Using similar triangles gives
$$x = \frac{27}{5}, \quad y = \frac{36}{5}.$$
Then
$$PA^2 = (10 - y)^2 + (15 - x)^2 = 100,$$
so $PA = 10$.

Answer: E) None of these

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45. What is the area of a $72°$ sector of a circle with diameter 30?

    (**A**) 15π (**B**) 30π (**C**) 45π (**D**) 60π (**E**) None of these

Solution: Radius $= 15$.
$$A = \frac{72}{360}\pi r^2 = \frac{1}{5}\pi(15^2) = 45\pi.$$

Answer: C) 45π

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46. A cube is inscribed in a sphere of radius r. Find the ratio of the sphere's volume to the cube's volume.

    (**A**) $\dfrac{\pi\sqrt{3}}{5}$ (**B**) $\dfrac{\pi\sqrt{3}}{4}$ (**C**) $\dfrac{\pi\sqrt{3}}{3}$ (**D**) $\dfrac{\pi\sqrt{3}}{2}$ (**E**) None of these

Solution: Space diagonal of cube $= 2r$. Cube side $s = \dfrac{2r}{\sqrt{3}}$.
$$V_{\text{cube}} = s^3 = \frac{8r^3}{3\sqrt{3}}, \quad V_{\text{sphere}} = \frac{4}{3}\pi r^3.$$
Thus ratio:
$$\frac{\dfrac{4}{3}\pi r^3}{\dfrac{8r^3}{3\sqrt{3}}} = \frac{\sqrt{3}\pi}{2}.$$

47. In $\triangle ABC$, the point D is on $\overline{AB}$ and the point E is on $\overline{BC}$. Suppose $AC = 39$, $DB = 9$, $DE = 13$, $AD = BE$, and $m\angle BDE = m\angle BCA$. Find the perimeter of $\triangle BDE$.

(**A**) $\dfrac{49}{2}$ (**B**) $\dfrac{53}{2}$ (**C**) $\dfrac{57}{2}$ (**D**) $\dfrac{61}{2}$ (**E**) None of these

Solution: Triangles ABC and BDE are similar. Let $AD = BE = x$.

$$\frac{x+9}{39} = \frac{x}{13} \Rightarrow x = \frac{9}{2}.$$
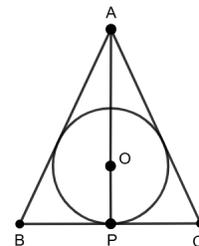
Thus

$$\text{Perimeter} = 9 + \frac{9}{2} + 13 = \frac{53}{2}.$$

Answer: B) $\dfrac{53}{2}$

48. A circle is inscribed in isosceles $\triangle ABC$, with $AC = AB = 12$ and $BP = 4$ where P is tangent point on $\overline{BC}$. Find the diameter of the circle.

(**A**) $4\sqrt{2}$ (**B**) $6\sqrt{2}$ (**C**) $8\sqrt{2}$ (**D**) $10\sqrt{2}$ (**E**) None of these

Solution: Let inradius be r. Right triangles formed give

$$(8\sqrt{2})^2 - 8^2 = r^2 \Rightarrow r^2 = 8.$$

Thus diameter $= 2r = 4\sqrt{2}$.

Answer: A) $4\sqrt{2}$

49. Square $ABCD$ has side length 1. Point E is on $\overline{CD}$ and F is inside so that $\overline{EF} \perp \overline{DC}$ and $AF = BF = EF$. Find the area of $\triangle AFB$.

(**A**) $\frac{1}{16}$ (**B**) $\frac{1}{8}$ (**C**) $\frac{3}{16}$ (**D**) $\frac{1}{4}$ (**E**) None of these

Solution: Let $EF = x$ and $FG = 1 - x$. Using Pythagorean relations:
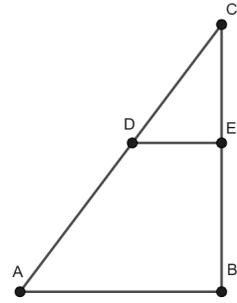
$$\left(\frac{1}{2}\right)^2 + (1-x)^2 = x^2 \Rightarrow 8x = 5 \Rightarrow x = \frac{5}{8}.$$

Then

$$\text{Area} = \frac{1}{2}(1)\left(\frac{3}{8}\right) = \frac{3}{16}.$$

Answer: C) $\dfrac{3}{16}$

50. In $\triangle ABC$, $m\angle C = 90°$, $DE \parallel AC$, $AC = 6$, $BC = 9\frac{5}{6}$, and $BD = \frac{28}{3}$. Find BE.

(**A**) $3\sqrt{5}$ (**B**) $4\sqrt{5}$ (**C**) $5\sqrt{5}$ (**D**) $6\sqrt{5}$ (**E**) None of these

Solution: First compute

$$AB = \sqrt{6^2 + \left(9\frac{5}{6}\right)^2} = 21.$$

Similar triangles give

$$\frac{BD}{AB} = \frac{BE}{BC} \quad \Rightarrow \quad \frac{28/3}{21} = \frac{BE}{59/6}.$$

Thus $BE = 4\sqrt{5}$.

Answer: B) $4\sqrt{5}$